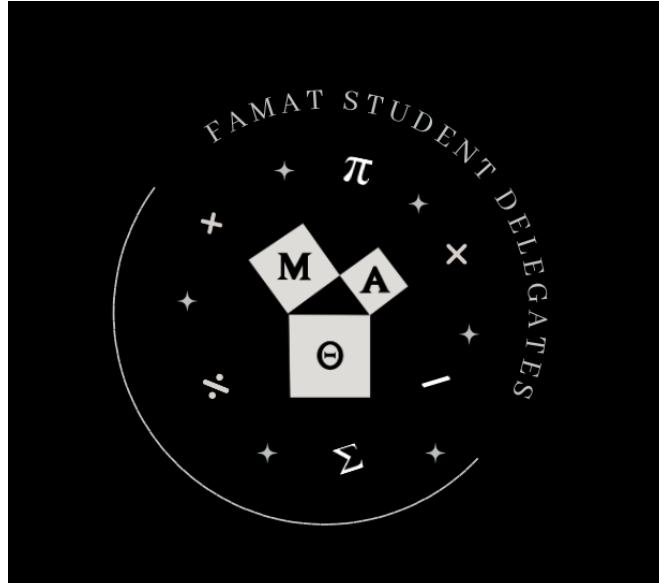


# 2023 Student Delegate Invitational

## Theta Individual



- You may not leave the zoom until the testing time of one hour is complete.
- No calculators or any other electronic devices are permitted.
- Sunglasses and hats are not to be worn.
- At the end of the testing session, turn in only the google form. You may use whatever scrap paper you wish, provided it is blank.
- The scoring will be 5 points for each question answered correctly, 1 point for each question left blank, and 0 points for each missed question. Ties will be broken using the sudden death method. Ask your proctor if you do not know what this is.
- Answers will be posted at the end of the test.
- If you believe that none of the given answers are correct, choose answer choice E, for none of the above.
- If you believe that multiple answer choices are correct, choose one and file a dispute after the test.
- Unless a question asks for an approximation or a rounded answer, give the exact answer.
- You will have 60 minutes to complete this test, with warnings given when you have 15 minutes, 5 minutes, and 1 minute left.

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- As Jerry is running around in a circle, he is yelling out numbers. Tom is taking those numbers and combining them. Jerry is under the impression that Tom is adding, while Tom is really multiplying. What is the positive difference between Tom and Jerry's final numbers if Jerry yells out 3, 5, 8, 5, 2, 1?  
(A) 0 (B) 376 (C) 776 (D) 824 (E) NOTA
- Omer needs an "A" (defined as anything greater than or equal to a 90%) for his final calculus grade. If he got a 80% for his last two test grades that account for 60% of his grade, what is the lowest grade he must get to score an "A" on his overall grade on a homework assignment that will be placed in the 40% weighted category?  
(A) 94 (B) 96 (C) 98 (D) 100 (E) NOTA
- Maria wants to take a picture of her 5 friends. She wants them all standing in a line shoulder-to-shoulder. If she wants the tallest in the middle but does not care about where anyone else goes, how many different ways can she arrange her friends? Assume no friends are the same height.  
(A) 4 (B) 6 (C) 24 (D) 120 (E) NOTA
- If  $\frac{30X}{50Y} = 3$ , what is the value of  $\frac{6X - 40Y}{10Y}$ ?  
(A) -6 (B) -1 (C) 1 (D) 6 (E) NOTA
- Given that  $\log_2 7$  is 2.807, simplify  $\log_{32} 28$  to the nearest thousandth.  
(A) 0.961 (B) 0.561 (C) 3.207 (D) 3.082 (E) NOTA
- A square is inscribed in a circle of radius 10. What is the area of ONE of the regions between the circle and the square?  
(A)  $100\pi - 25$  (B)  $100\pi - 200$  (C)  $25\pi - 50$  (D)  $25\pi - 25$  (E) NOTA
- Aadi draws a trapezohedron with a volume of  $1 \text{ cm}^3$ . He then thinks it is too small, so he increases its side lengths by a factor of 4. What is the volume of the new shape?  
(A) 4 (B) 8 (C) 20 (D) 64 (E) NOTA
- Find  $\sum_{x=5}^{\infty} \frac{4}{x^2 - 7x + 12}$   
(A) 8 (B) 4 (C) 2 (D) 1 (E) NOTA
- How many solutions does the following equation have?  $x - \frac{7}{x-3} = 3 - \frac{7}{x-3}$   
(A) infinitely many (B) one solution (C) two solutions (D) no solutions (E) NOTA
- Given the equation  $x^2 + ax + b$  has distinct roots  $a$  and  $b$ , what is the value of  $3b + 8a$ ?  
(A) -1 (B) 2 (C) 4 (D) 1 (E) NOTA
- The polynomial  $x^3 - 1$  has roots  $a$ ,  $b$ , and  $c$ , and the polynomial  $x^2 - 1$  has roots  $p$  and  $q$ .  $P(x)$  is a sixth-degree polynomial with roots  $a + p$ ,  $a + q$ ,  $b + p$ ,  $b + q$ ,  $c + p$ , and  $c + q$ . What is  $P(1)$ ?  
(A) -1 (B) 0 (C) 7 (D) -7 (E) NOTA

12. Suppose the medians that intersect the equivalent sides of an isosceles triangle intersect at right angles. If the length of the other side of the triangle is  $4\sqrt{2}$ , what is the area of the triangle?
- (A) 8 (B) 16 (C) 24 (D) 48 (E) NOTA
13. What is the area of a triangle with side lengths 17 cm, 14 cm, and 9 cm?
- (A) Can't be solved (B)  $\sqrt{3689}$  cm<sup>2</sup> (C)  $6\sqrt{110}$  cm<sup>2</sup> (D) 63 cm<sup>2</sup> (E) NOTA
14. A box contains 8 red marbles, 7 green marbles, and 5 blue marbles. If two marbles are drawn at random in succession without replacement, what is the probability that neither is blue?
- (A)  $\frac{9}{16}$  (B)  $\frac{21}{38}$  (C)  $\frac{113}{76}$  (D)  $\frac{3}{2}$  (E) NOTA
15. For  $x > 1$ , what is the sum of all values  $x$  in which  $\frac{6}{x}$  is an integer?
- (A) 5 (B) 11 (C) 13.7 (D) 14.7 (E) NOTA
16. Simplify:  $(i - i^{-1})^{-1}$
- (A)  $\frac{-i}{2}$  (B)  $\frac{-i}{4}$  (C)  $\frac{i}{2}$  (D)  $\frac{i}{4}$  (E) NOTA
17. What is the area of the largest square that can be inscribed in an ellipse with equation  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ ?
- (A)  $\frac{25}{144}$  (B)  $\frac{576}{25}$  (C)  $\frac{144}{25}$  (D)  $\frac{25}{576}$  (E) NOTA
18. What are the last two digits of  $7^{2023}$ ?
- (A) 01 (B) 07 (C) 43 (D) 49 (E) NOTA
19. Given  $\log(2) = p$  and  $\log(3) = q$ , what is the value of  $\log(375)$  in terms of  $p$  and  $q$ ?
- (A)  $3pq - q$  (B)  $\frac{3-p}{q}$  (C)  $3pq + q$  (D)  $q - 3p + 3$  (E) NOTA
20. For all  $a \neq b$ , define the operation  $a \star b$  as follows:  
 Consider the graph of the function  $f(x) = x^3 - 3x + 1$ . The points  $(a, f(a))$  and  $(b, f(b))$  lie on this graph, and the line connecting these points also intersects  $f(x)$  at a third point.  $a \star b$  is defined to be the  $x$ -coordinate of this third point.  
 $a \star b$  is undefined for  $a = b$ . Find the value of  $n$  such that
- $$(n \star 2n) \star (2n \star 3n) = 2024.$$
- (A) -253 (B) -179 (C) 253 (D) 278 (E) NOTA
21. How many distinct ways can you rearrange the word "FORTNITE"?
- (A) 20160 (B) 40320 (C) 5040 (D) 1 (E) NOTA
22. Vishwa is struggling with building muscle, so he has started using protein powder. He is debating between 2 different types. Type A has the same grams of protein per serving as the  $y$ -coordinate of the  $y$ -intercept of  $50(y - 5) = 25x + 2000$ . Type B has the same grams of protein per serving as the  $y$ -coordinate of the vertex of  $f(x) = -5x^2 + 30x + 1$ . Each type comes with 50 servings in the container. What is the positive difference between the total grams of protein in each container?
- (A) 15 (B) 1 (C) 12 (D) 4 (E) NOTA

23. What is the real part of the complex number  $(4 - 3i)^3$ ?
- (A) -32                      (B) -55                      (C) 12                      (D) -45                      (E) NOTA
24. The Absurdly Arranged Chocolate Company packages chocolates in unusual shapes and quantities. Their most recent product, the Deluxe Chocolate Pyramid, features a massive stack of  $70^2$  spherical chocolates, each one centimeter in diameter, packed together in a square pyramidal structure. The chocolates are packaged in a similarly square pyramidal box of negligible thickness that touches every chocolate on the outer faces of the pyramid. What is the height of the box?
- (A)  $4\sqrt{2} + 9$                       (B)  $4\sqrt{3} - 9$                       (C)  $12\sqrt{3} + 12$                       (D)  $12\sqrt{2} + 12$                       (E) NOTA
25. What is the determinant of the following matrix?  $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 3 \\ 1 & 5 & 4 \end{bmatrix}$
- (A) 14                      (B) 23                      (C) 18                      (D) 25                      (E) NOTA
26. What is the determinant of the resulting matrix?  $\begin{bmatrix} 10 & 5 & 2 \\ 0 & 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ 2 & 20 \\ 0 & 6 \end{bmatrix}$
- (A) 0                      (B) 1800                      (C) 6600                      (D) 10320                      (E) NOTA
27. An icosahedron is a regular polyhedron with 20 faces, each in the shape of an equilateral triangle. For an icosahedron of edge length 2 centered on the origin, the coordinates of the vertices that make up one of its triangular faces can be written as  $(0, 1, \phi)$ ,  $(1, \phi, 0)$ , and  $(\phi, 0, 1)$ , where  $\phi = \frac{1+\sqrt{5}}{2}$  is the golden ratio. What is the volume of this icosahedron?
- (A)  $\frac{30+10\sqrt{5}}{3}$                       (B)  $\frac{30-10\sqrt{5}}{3}$                       (C)  $\frac{15+5\sqrt{3}}{3}$                       (D)  $\frac{15-5\sqrt{3}}{3}$                       (E) NOTA
28. Angles  $\angle ABC$  and  $\angle DEF$  are complementary, with  $\angle ABC = 6x + 12$  and  $\angle DEF = 8x - 20$ . What is the measure of the supplement of  $\angle ABC$ ?
- (A) 46                      (B) 54                      (C) 132                      (D) 126                      (E) NOTA
29. Manas, Yash, Diya, Tarang, Sahiti, Hemangi, Kayomi, Ishaan, Aaditya, and Sathwik are trying to play a game of Ultimate Frisbee. How many unique teams of 5 can they generate from the given people?
- (A) 252                      (B) 720                      (C) 5040                      (D) 30240                      (E) NOTA
30. How many ways can you organize the names from question 29 so that they are in alphabetical order?
- (A) 1                      (B)  $10!$                       (C)  $\frac{10!}{2}$                       (D) Not possible                      (E) NOTA