For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Solve the following equation for x: $\dfrac{3}{5} + \dfrac{1}{2}(3x + 6) = 2x$

    (A) $\dfrac{18}{5}$ (B) 6   (C) $\dfrac{36}{5}$ (D) 12   (E) NOTA

2. What is the x-intercept of the line through the point (4,4) that is perpendicular to the line with equation $4x - 2y = 8$

    (A) 6   (B) 8   (C) 10   (D) 12   (E) NOTA

3. The complement of an angle is 15 degrees less than three times the angle. What is the measure of four times the angle?

    (A) $75°$ (B) $90°$ (C) $105°$ (D) $195°$ (E) NOTA

4. What value of k will cause the following quadratic to have a discriminant of 0:

    $x^2 - 16x + (2k + 4) = 0$

    (A) -34 (B) 30   (C) 64   (D) 126   (E) NOTA

5. An equilateral triangle with side length s is inscribed within a circle with area 4π. What is the value of s:

    (A) $\dfrac{\sqrt{3}}{3}$ (B) $\dfrac{2\sqrt{3}}{3}$ (C) $\dfrac{3\sqrt{3}}{2}$ (D) $2\sqrt{3}$ (E) NOTA

6. The function $f(x) = \dfrac{2x^2 + 6x}{x^2 - 4}$ has a horizontal asymptote at $y = a$ and two vertical asymptotes at $x = b$ and $x = c$. What is the value of $a \cdot b \cdot c$?

    (A) -8 (B) -4 (C) 2   (D) 4   (E) NOTA

7. Find the distance between the centers of the following two ellipses:

    $x^2 - 16x + y^2 + 4y + 32 = 0$

    $25x^2 - 50x + 16y^2 - 32y - 359 = 0$

    (A) 5   (B) $\sqrt{50}$ (C) $\sqrt{58}$ (D) 8   (E) NOTA

8. Find the value of $f^{-1}(x)$ for the function $f(x) = \dfrac{3x + 2}{2x - 1}$

    (A) $\dfrac{x + 2}{2x - 3}$ (B) $\dfrac{x - 2}{2x + 3}$ (C) $\dfrac{2x - 1}{3x + 2}$ (D) $\dfrac{2x + 1}{3x - 2}$ (E) NOTA

9. If Sharvaa practices the guitar for t hours, his skill level is represented by $S(t) = -3x^2 + 12x + 10$. What is the greatest skill level he can ever reach?

    (A) 10   (B) 19   (C) 22   (D) 25   (E) NOTA

10. The function $f(x) = x^3 - 31x + 30 = 0$ has one solution at $x = 1$. If the other two solutions are $x = a$ and $x = b$ such that $a > b$, what is the value of $a - b$.

    (A) 11   (B) 13   (C) 19   (D) 24   (E) NOTA

11. What is the set of all values for which the function $f(x) = \sqrt{x^2 + x - 2}$ is undefined?

    (A) $(-\infty, \infty)$ (B) $(-1, 2)$ (C) $[-2, 3)$ (D) $(-2, 1)$ (E) NOTA

12. Find the sum of the solutions to $\log_2(x-6) + \log_2(x-8) = 3$

    (A) 4   (B) 8   (C) 10   (D) 14   (E) NOTA

13. Find the sum of all possible integer values for the third side of a triangle which has two sides of length 7 and 10:

    (A) 110   (B) 130   (C) 147   (D) 159   (E) NOTA

14. Sharvaa has \$100 when he enters the FAMAT merchandise store. He finds a shirt that costs \$100, but discovers there is an additional 20% discount on all items at the store. When he arrives at the cash register he realizes that he has to pay a 10% sales tax on the price after discount. How much money does Sharvaa have after buying the shirt?

    (A) \$30   (B) \$20   (C) \$12   (D) \$10   (E) NOTA

15. What is the value of $\log_2(49) \cdot \log_9(8) \cdot \log_7(3)$

    (A) $\log_3(14)$ (B) 2   (C) 3   (D) $\log_2(21)$ (E) NOTA

16. Given that $f(x^2 + 2x + 3) = x^4 + 5x^3 - x^2 - 22x - 5$, what is the value of $f(11)$

    (A) -429 (B) 3   (C) 273   (D) 20928   (E) NOTA

17. Find $X + Y + Z$ given the following:

$$X + Y - 2Z = -1$$
$$2X - 3Y + Z = -8$$
$$3X - 2Y + 4Z = 10$$

    (A) 10   (B) 15   (C) 19   (D) 31   (E) NOTA

18. Sharvaa needs help solving his triangular troubles. He has two triangles, but isn't able to prove that they are similar. Which of the following methods will **not** help him escape this crisis.

    (A) SAS   (B) SSS   (C) AA   (D) SSA   (E) NOTA

19. Tanmay and Himal plan to meet at Jane Street some time between 4pm and 5pm. They will both arrive at random times within the hour. Himal will stay for 10 minutes. However, Tanmay is busy grading statistics disputes so he will only be able to stay for 5 minutes. What is the probability that they will meet during that hour?

    (A) $\dfrac{19}{96}$ (B) $\dfrac{67}{288}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{91}{288}$ (E) NOTA

20. If $2 \cdot 8^{2-3x} = \frac{1}{32}$, then give the value of $6x - 11$:

    (A) -10 (B) 0   (C) 5   (D) 21   (E) NOTA

21. Tanmay is in dire need of a suitor. Thankfully, the FAMAT Delegates are here to help. Himal uses her abundant experience to find 1 suitors every 6 day, while Jesse uses the FAMAT dating app to find 5 suitors every 2 day. Working together, how many hours will it take them to find 1 suitor for Tanmay.

    (A) 3   (B) $\frac{8}{3}$ (C) 9   (D) 16   (E) NOTA

22. The roots $r > s > t$ of a cubic Polynomial $P(x) = x^3 - 15x^2 + 66x - 80$ form an arithmetic sequence. What is the value of r? Hint: Use Vieta's formulas.

    (A) -2   (B) 4   (C) 6   (D) 8   (E) NOTA

23. If $x^2 + \dfrac{1}{x^2} = 7$, what is the value of $x^6 + \dfrac{1}{x^6}$?

    (A) 21   (B) 23   (C) 161   (D) 322   (E) NOTA

24. The equation $\sqrt{x^2 + 4} = x + \sqrt{-8x + 4}$ has two solutions when solved over the real numbers. The smaller solution lies within which of the following ranges?

    (A) $[-3, -2)$ (B) $[-2, -1)$ (C) $[-1, 0)$ (D) $[2, 3)$ (E) NOTA

25. Srijan flips a fair coin 8 times. What is the probability that he gets more heads than tails?

    (A) $\dfrac{93}{256}$ (B) $\dfrac{117}{256}$ (C) $\dfrac{4}{9}$ (D) $\dfrac{1}{2}$ (E) NOTA

26. Let r, s, t be three roots of $x^3 + 12x^2 - 30x + 15 = 0$. Compute $r^2s + rs^2 + r^2t + rt^2 + s^2t + st^2$

    (A) 405   (B) 450   (C) 360   (D) 480   (E) NOTA

27. Positive reals x, y, z satisfies

    $$\log_x y + \log_y z + \log_z x = 10$$
    $$\log_y x + \log_z y + \log_x z = -12$$

    The only positive value among $\log_x y, \log_y z, \log_z x$ can be written in the form of $\frac{a+b\sqrt{b}}{c}$, where a, b, c are positive integers and the fraction cannot be simplified further. What is $a + b + c$?

    (A) 12   (B) 18   (C) 71   (D) 123   (E) NOTA

28. Let $P(x)$ be a polynomial which leaves a remainder of 8 when divided by $x - 2$ and a remainder of -4 when divided by $x + 2$. When $P(x)$ is divided by $x^2 - 4$ it leaves a remainder of $Q(x)$. What is the value of $Q(1)$

    (A) -3   (B) 5   (C) 10   (D) 15   (E) NOTA

29. A square $ABCD$ has point E, F lies on sides BC and CD. Given that $\angle EAF = 45°$ and $BE = 8, DF = 10$, find the length of EF:

    (A) $8\sqrt{2}$ (B) 6   (C) $10\sqrt{2}$ (D) 18   (E) NOTA

30. Congratulations on making it to question 30! All you have to do is unscramble "LTDAUTEI" and select what the first letter should be. Hint: I really love triangles.

    (A) A   (B) E   (C) L   (D) D   (E) NOTA