

Student Delegates Integration Bee Finals

Student Delegates

April 15, 2023

Introduction

Welcome everyone to the 3rd Student Delegates Integration Bee Finals! We are so glad that you chose to participate in this event!

Logistics

The first 3 elimination rounds will be best of 3, and the final elimination round will be best of 5. If there is no winner after 3/5 integrals, it will turn into a sudden death.

Let's Begin!

Integral 1

$$\int_0^1 2023x^{118} dx$$

Integral 1 Answer

17

Integral 3

$$\int (14x^6 + 8x^3) \sqrt[3]{x^7 + x^4} dx$$

Integral 3 Answer

$$\frac{3}{2}(x^7 + x^4)^{\frac{4}{3}} + C$$

Integral 4

$$\int_0^{\infty} e^{-3x} \cos(4x) dx$$

Integral 4 Answer

$$\frac{3}{25}$$

Integral 5

$$\int_{-\pi}^{\pi} \frac{x^6 \cos(x^2 + x^4 + x^6)}{2x + 4x^3 + 6x^5} dx$$

Integral 5 Answer

0

Integral 6

$$\int_1^2 (x + 1)\sqrt{x - 1} dx$$

Integral 6 Answer

$$\frac{26}{15}$$

Integral 7

$$\int_0^{\infty} \frac{1}{1+x^4} dx$$

Integral 7 Answer

$$\frac{\pi}{2\sqrt{2}}$$

Integral 8

$$\int_0^{100} e^{x - [x]} dx$$

Integral 8 Answer

$$100e - 100$$

Integral 9

$$\int_0^{\frac{\pi}{2}} \frac{1}{2 + \cos(x)} dx$$

Integral 9 Answer

$$\frac{\pi}{3\sqrt{3}}$$

Integral 10

$$\int_0^{\frac{\pi}{4}} \sqrt{\sec(x)} \tan(x) dx$$

Integral 10 Answer

$$2(\sqrt[4]{2} - 1)$$

Integral 11

$$\int_0^{\frac{\pi}{2}} x \sin(x) dx$$

Integral 11 Answer

1

Integral 11

$$\int \frac{\sin(x) + \cos(x)}{3 \sin(x) + 4 \cos(x)} dx$$

Integral 11 Answer

$$\frac{7x - \ln(3 \sin(x) + 4 \cos(x))}{25} + C$$

Integral 12

$$\int_0^{\sqrt{7}} \sqrt[3]{x^5 + x^3} dx$$

Integral 12 Answer

$$\frac{45}{8}$$

Integral 13

$$\int_1^2 \frac{x-1}{x^2 \ln(x) + x} dx$$

Integral 13 Answer

$$\ln\left(\frac{1}{2} + \ln(2)\right)$$

Integral 14

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2(x)}{\sin(x) + \cos(x)} dx$$

Integral 14 Answer

$$\frac{\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)}{2\sqrt{2}}$$

Integral 15

$$\int_0^{\infty} \frac{1}{(1+x^2)(1+x^e)} dx$$

Integral 15 Answer

$$\frac{\pi}{4}$$

Integral 16

$$\int_1^e \frac{3 \ln^2(x) - 4 \ln^3(x)}{x^5} dx$$

Integral 16 Answer

$$\frac{1}{e^4}$$

Integral 17

$$\lim_{n \rightarrow \infty} \int_0^{\frac{\pi}{4}} \sqrt[n]{\sin^n(x) + \cos^n(x)} dx$$

Integral 17 Answer

$$\frac{\sqrt{2}}{2}$$

Integral 18

$$\int_0^{\infty} \frac{\arctan(x)}{x^2 + \sqrt{3}x + 1} dx$$

Integral 18 Answer

$$\frac{\pi^2}{12}$$

Integral 19

$$\int_{-2}^2 | \lfloor x \rfloor - 2 | dx$$

Integral 19 Answer

10

Integral 20

$$\int_0^2 \frac{\sqrt{x+1}}{\sqrt{x+1} + \sqrt{3-x}} dx$$

Integral 20 Answer

1

Integral 21

$$\int_0^{\infty} x^{-\pi \ln(x)} dx$$

Integral 21 Answer

$$\sqrt[4\pi]{e}$$

Integral 22

$$\int_0^{\infty} e^{-2x} (\lfloor x \rfloor + 1) dx$$

Integral 22 Answer

$$\frac{e^2}{2(e^2 - 1)}$$

Integral 23

$$\int_0^{\infty} \frac{1 - 4^x}{x8^x} dx$$

Integral 23 Answer

$$\ln\left(\frac{1}{3}\right)$$

Integral 24

$$\int_0^{\infty} e^{-x^2} \cos(5x) dx$$

Integral 24 Answer

$$\frac{\sqrt{\pi}}{2} e^{-\frac{25}{4}}$$

Integral 25

$$\lim_{n \rightarrow \infty} n \int_n^{2n} \frac{1}{(x+n)^2} dx$$

Integral 25 Answer

$$\frac{1}{6}$$

Integral 26

$$\int \frac{\csc^2(x) + 4x^2}{(4x^2 + 1)(\cot^2(x))} dx$$

Integral 26 Answer

$$\frac{1}{2} \arctan(2x) + \tan(x) - x + C$$

Integral 27

$$\int_0^{\infty} e^{-x}(x^3 + 3x^2 + 3x + 1) dx$$

Integral 27 Answer

16

Integral 28

$$\lim_{n \rightarrow \infty} \frac{(-2)^n}{(n+1)!} \int_0^1 \ln(x)^{n+1} \ln(1-x) dx$$

Integral 28 Answer

$$\frac{1}{4}$$

Integral 29

$$\int_0^{2023} \left[\left(x + \frac{1}{x} \right)^2 \right] - \left[\left(x - \frac{1}{x} \right)^2 \right] dx$$

Integral 29 Answer

8092

Integral 30

$$\int_0^{\frac{\pi}{2}} x^2 \cos(x) dx$$

Integral 30 Answer

$$\frac{\pi^2}{4} - 2$$

Integral 31

$$\int_0^{\infty} \frac{x}{\sqrt{e^{2x} - 1}} dx$$

Integral 31 Answer

$$\frac{\pi \ln(2)}{2}$$

Integral 32

$$\int_0^{\ln(5)} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx$$

Integral 32 Answer

$$4 - \pi$$

Integral 33

$$\int_0^{\frac{\pi}{3}} \sec^6(x) dx$$

Integral 33 Answer

$$\frac{24\sqrt{3}}{5}$$

Integral 34

$$\int_0^1 \frac{4x^2 + 4x + 4}{4x^3 + 6x^2 + 12x + 22} dx$$

Integral 34 Answer

$$\frac{\ln(2)}{3}$$

Integral 35

$$\int e^x \left(\ln(x) + \frac{3}{x} - \frac{3}{x^2} + \frac{2}{x^3} \right) dx$$

Integral 35 Answer

$$e^x \left(\ln(x) + \frac{2}{x} - \frac{1}{x^2} \right) + C$$

Integral 36

$$\int_{4306}^{4316} x \, dx$$

Integral 36 Answer

43110

Integral 37

$$\int_0^{\frac{\pi}{4}} x \tan^2(x) dx$$

Integral 37 Answer

$$\frac{\pi}{4} - \frac{\pi^2}{32} - \frac{\ln(2)}{2}$$

Integral 38

$$\int_{-1}^0 \frac{\ln(1-x)}{x} dx$$

Integral 38 Answer

$$-\frac{\pi^2}{12}$$

Integral 39

$$\int_0^{2\pi} e^{\cos(x)} \cos(\sin(x)) dx$$

Integral 39 Answer

$$2\pi$$

Integral 40

$$\int \frac{2023^{\sqrt{x}}}{\sqrt{x}} dx$$

Integral 40 Answer

$$2 \frac{2023^{\sqrt{x}}}{\ln(2023)} + C$$

Integral 41

$$\int_0^1 \frac{x^2 - 1}{\ln(x)} dx$$

Integral 41 Answer

$$\ln(3)$$

Integral 42

$$\int_0^1 \frac{1 + x^2 + x^4 + x^8}{1 + x^4} dx$$

Integral 42 Answer

$$\frac{1}{5} + \frac{\pi\sqrt{2}}{4}$$

Integral 43

$$\int_0^{\pi} \sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{2n+1} dx$$

Integral 43 Answer

$$\frac{\pi^2}{4}$$

Integral 44

$$\int_0^{2\pi} \sin(2x) \sin(3x) dx$$

Integral 44 Answer

0

Integral 45

$$\int_0^2 \binom{x}{3} dx$$

Integral 45 Answer

0

Integral 46

$$\int \frac{\sqrt{4 - x^2}}{x^2} dx$$

Integral 46 Answer

$$-\frac{\sqrt{4-x^2} + x \arcsin\left(\frac{x}{2}\right)}{x} + C$$

Integral 47

$$\int_0^{\infty} x^6 e^{-x} dx$$

Integral 47 Answer

720

Integral 48

$$\int_0^{\infty} \frac{e^{\frac{1}{1+x^2}} \cos\left(\frac{x}{1+x^2}\right)}{1+x^2} dx$$

Integral 48 Answer

$$\frac{\sqrt{e\pi}}{2}$$

Integral 49

$$\int_0^1 \ln(1 + x^2) dx$$

Integral 49 Answer

$$\frac{\pi}{2} + \ln(2) - 2$$

Integral 50

$$\int_{\sqrt{3}}^{3\sqrt{3}} \frac{\ln(x)}{x^2 + 9} dx$$

Integral 50 Answer

$$\frac{\pi \ln(3)}{18}$$

Integral 51

$$\int_1^{e^{\frac{\pi}{2}}} \cos^2(\ln(x)) dx$$

Integral 51 Answer

$$\frac{2e^{\frac{\pi}{2}} - 3}{5}$$

Integral 52

$$\int_0^1 \arctan\left(\frac{1}{x^2 - x + 1}\right) dx$$

Integral 52 Answer

$$\frac{\pi}{2} - \ln(2)$$

Integral 53

$$\int_0^e W(x) dx$$

1

¹ $W(x)$ is the inverse of xe^x

Integral 53 Answer

$$e - 1$$

Integral 54

$$\int_0^{\infty} \frac{\sin(x)}{\sqrt{x}} dx$$

Integral 54 Answer

$$\sqrt{\frac{\pi}{2}}$$

Integral 55

$$\int_0^{\infty} x^4 e^{-x^2} dx$$

Integral 55 Answer

$$\frac{3\sqrt{\pi}}{8}$$

Integral 56

$$\int \sec^5(x) dx$$

Integral 56 Answer

$$\frac{1}{4} \sec^3(x) \tan(x) + \frac{3}{8} \sec(x) \tan(x) + \frac{3}{8} \ln |\sec(x) + \tan(x)| + C$$

Integral 57

$$\int_0^{\sqrt{2}} \{x^2\} dx$$

Integral 57 Answer

$$1 - \frac{\sqrt{2}}{3}$$

Integral 58

$$\int_0^1 \frac{\ln^2(x)}{\sqrt{x(1-x)}} dx$$

Integral 58 Answer

$$\frac{\pi^3}{3} + 4\pi \ln^2(2)$$

Integral 59

$$\int \frac{x^2 + 2x \ln(x) - x}{x^2 + 2x \ln(x) + \ln^2(x)} dx$$

Integral 59 Answer

$$\frac{x^2}{x + \ln(x)} + C$$

Integral 60

$$\int_0^{\infty} \ln\left(\frac{e^x}{e^x - 1}\right) dx$$

Integral 60 Answer

$$\frac{\pi^2}{6}$$

Integral 61

$$\int_{-\infty}^{\infty} \frac{x^2(x^2 - 1)^2}{(x^2 - 1)^4 + x^4} dx$$

Integral 61 Answer

$$\frac{\pi}{\sqrt{2}}$$

Integral 62

$$\int \frac{e^{3x}}{\sqrt{1 - e^{2x}}} dx$$

Integral 62 Answer

$$\frac{1}{2} \arcsin(e^x) - \frac{1}{2} e^x \sqrt{1 - e^{2x}} + C$$

Integral 63

$$\lim_{n \rightarrow 0} \frac{1}{n^3} \int_0^n \ln\left(\frac{x}{\sin(x)}\right) dx$$

Integral 63 Answer

$$\frac{1}{18}$$

Integral 64

$$\int \frac{x^3}{1 + x + \frac{x^2}{2} + \frac{x^3}{6}} dx$$

Integral 64 Answer

$$6x - 6 \ln\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) + C$$

Integral 65

$$\int \frac{x^3}{1 + x + \frac{x^2}{2} + \frac{x^3}{6}} dx$$

Integral 65 Answer

$$\frac{3\pi}{8}$$

Integral 66

$$\int_0^{\infty} x \sin(2x) e^{-3x} dx$$

Integral 66 Answer

$$\frac{4}{25}$$

Integral 67

$$\int_0^{\pi} \ln(5 + 3 \cos(x)) dx$$

Integral 67 Answer

$$\pi \ln\left(\frac{9}{2}\right)$$

Integral 68

$$\int_0^4 \sqrt{\frac{4-x}{4+x}} + \sqrt{\frac{4+x}{4-x}} dx$$

Integral 68 Answer

$$4\pi$$

Integral 69

$$\int_0^{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n \sin((2n+1)x)}{(2n+1)^2} dx$$

Integral 69 Answer

$$\frac{\pi^3}{16}$$

Integral 70

$$\int_0^1 x \sqrt{x} \sqrt[3]{x} \sqrt[4]{x} \dots dx$$

Integral 70 Answer

$$\frac{1}{e}$$