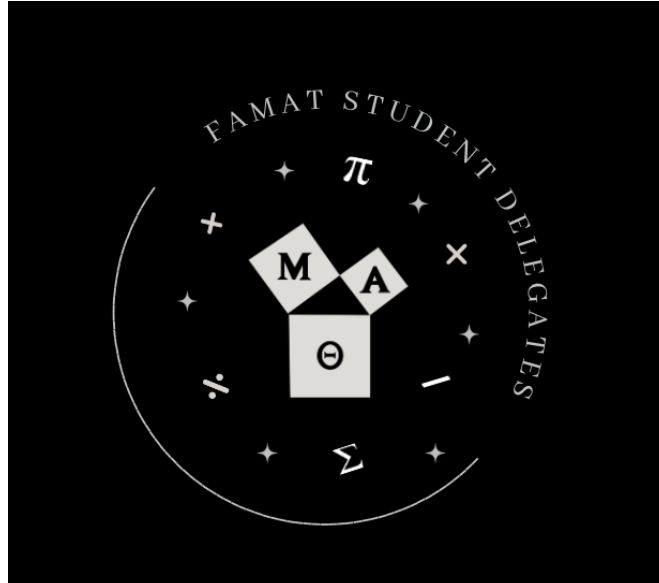# 2023 Student Delegate Invitational

Statistics Individual

- You may not leave the zoom until the testing time of one hour is complete.
- Sunglasses and hats are not to be worn.
- At the end of the testing session, turn in only the google form. You may use whatever scrap paper you wish, provided it is blank.
- The scoring will be 5 points for each question answered correctly, 1 point for each question left blank, and 0 points for each missed question. Ties will be broken using the sudden death method. Ask your proctor if you do not know what this is.
- Answers will be posted at the end of the test.
- If you believe that none of the given answers are correct, choose answer choice E, for none of the above.
- If you believe that multiple answer choices are correct, choose one and file a dispute after the test.
- Unless a question asks for an approximation or a rounded answer, give the exact answer.
- You will have 60 minutes to complete this test, with warnings given when you have 15 minutes, 5 minutes, and 1 minute left.

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Commander G is not only a top statistician, he is also a world class linguist! In addition to being fluent in English, he can also speak Canadian and Spanish. On any given day, Commander G will speak in a random combination of these three languages (always speaking in at least one of them). Of the 100 days he was observed, Commander G spoke only English on 34 of them, only Spanish on 22 of them, and only Canadian on 8 of them. He spoke all three on 10 days. If he spoke Canadian on a total of 25 days, how many days did he speak only English and Spanish?

   (A) 19   (B) 21   (C) 24   (D) 26   (E) NOTA

2. Jin-Yang Li is a very talented mathematician. So talented in fact that almost every college wanted him! Jin got into Harvard, Brown, Yale, and Stanford but got rejected from Broward Community College. These schools will only accept or deny students, there are no other possible results. If these schools have respective acceptance rates of 4%, 7%, 5%, 6%, and 98%. What was the probability that Jin would have the acceptances that he did? (The answer is not 1.)

   (A) 0.000008   (B) 0.000000168   (C) 0.015945408   (D) Can't Be Determined   (E) NOTA

Use the following information for the next 4 questions. Unless specified otherwise, each question is independent of one another so the situations in one question will not affect the situations in another.

Reechard has a unique condition that causes him to grow continuously. The following data represents Reechard's height in inches and in how years he will achieve that height.

Height (inches)	Years in Future
65	10
70	20
75	30
80	40
85	50
110	60

3. Treat Reechards height as its own set of data. Using the 1.5 IQR rule, how many outliers does this data have?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

4. If we treat the number of years in the future as the explanatory variable and Reechard's height as the response variable, we can calculate a least squares regression line to fit our data! If we do that and round only our final values to 3 decimal places, what is the equation of our line, in slope-intercept form?

   (A) $y = 0.786x + 53.333$ (B) $y = 0.922x + 53.333$ (C) $\hat{y} = 0.786x + 53.333$ (D) $\hat{y} = 0.922x + 53.333$ (E) NOTA

5. Using the least squares regression line from the previous problem, which of the following is the value of the coefficient of determination, rounded to 3 decimal places?

   (A) -0.922   (B) -0.850   (C) 0.850   (D) 0.922   (E) NOTA

6. Using the same least squares regression line from problem 4, which of the following is the proper interpretation of the y-intercept?

   (A) Reechard's current height is 53.333 inches
   (B) If Reechard continues to grow forever, his ending height will be 53.333 inches
   (C) In 53.333 years, Reechard will stop growing
   (D) The y-intercept of a least squares regression line never has any meaning
   (E) NOTA

7. Nathan Wei just landed a job as the host of a brand new game show, it will premiere on LIVE TV in 3 days! As part of the game show, there are three closed doors. Behind one of the doors, is a stuffed monkey but the other two have nothing behind them. The first contestant on the show, Julius, gets to pick one of the three doors. Nathan will then open one of the other two doors to reveal that it is empty. Finally, Julius gets the opportunity to either stick with the door he picked originally or switch the other unopened one with the goal of selecting the door with the monkey behind it. Julius discovers that the probability of winning increases if he switches to the other unopened door! What famous problem does this scenario represent?

   (A) Catalan Numbers   (B) Maximizing Odds Game Show   (C) P=NP
   (D) Monty Hall   (E) NOTA

   Use the following information for the next 3 questions. Unless specified otherwise, each question is independent of one another so the situations in one question will not affect the situations in another.
   One of the biggest challenges of creating a formula for the annual FAMAT Student Delegate Squirty Chicken Awards is accounting for the difference in difficulty between tests throughout the year. For example, the scores on last years Precalculus Individual at Cypress Bay followed an approximately normal distribution with a mean of 31 and standard deviation of 29 while the Precalculus Individual at NSU followed an approximately normal distribution with a mean of 62 and standard deviation of 32. For the purpose of these questions, assume that the score a person gets is truly random and is not impacted by math ability or practice and that the results of competitions are completely independent of one another. Also, ignore scoring limitations of standard FAMAT exams.

8. What is the probability, rounded to the thousandths place, that a random FAMAT competitor, Meghna Mullangi, gets a score above 80 on the Cypress Bay test and a score between 70 and 90 on the NSU test?

   (A) 0.010   (B) 0.046   (C) 0.211   (D) 0.257   (E) NOTA

9. Mr. Yu's scores are impressively consistent across competitions. On the Cypress Bay test, he scored a 68. If his score on the NSU test is at the same percentile as his score on the Cypress Bay test, what did Mr. Yu get, rounded to the nearest integer?

   (A) 89   (B) 90   (C) 103   (D) 104   (E) NOTA

10. The scoring rules of FAMAT exams is changing this year! Instead of being +4 for correct answers, +0 for blank answers, and -1 for incorrect answers, it will now be +5 for correct answers, +1 for blanks answers, and +0 for incorrect answers. Assuming that every test in the Cypress Bay competition were re-scored to follow the new rules, which of the following would be the new mean and standard deviation? Answers are in the form (mean, standard deviation).

    (A) (31, 29)   (B) (61, 29)   (C) (61, 59)   (D) (92, 32)   (E) NOTA

11. Cyrus is a big gambler. He likes to play a modified version of poker where he randomly selects 5 cards from 2 identical decks, without replacement. What is the probability that on his last hand, he got a royal flush?

    (A) $\frac{1}{2}$ (B) $\frac{1}{21658}$ (C) $\frac{1}{10829}$ (D) $\frac{1920}{2299063}$ (E) NOTA

12. Team Unicorn and Team Buffalo are playing a game against each other. Each team has 7 players, the players of each team are placed in order of flexibility, with the most flexible being first. The winner of each round stays on to play the next player of the opposite team. For example, if the second player of Team Unicorn beats the fourth player of Team Buffalo, the next round would be the second player of Team Unicorn vs the fifth player of Team Buffalo. How many unique ways are there for the game to play out?

    (A) 49   (B) 3432   (C) 5040   (D) 25401600   (E) NOTA

Use the following information for the next 4 questions. Unless specified otherwise, each question is independent of one another so the situations in one question will not affect the situations in another.

There is a very special coin. On one side is a dragon and on the other is a knight. The probability of getting the dragon is 0.3, the probability of getting the knight is 0.65, and the probability of the coin landing on it's side is 0.05.

13. Eric Gao will keep flipping the coin until the coin lands on it's side. What is the expected number of flips he will have to make before the coin lands on it's side?

    (A) 20   (B) 15   (C) 10   (D) 5   (E) NOTA

14. Samer is going to flip the coin 120 times. What is the probability that it takes him exactly 15 flips to see his first dragon, rounded to 6 decimal places?

    (A) 0.995252   (B) 0.002035   (C) 0.000004   (D) 0.000001   (E) NOTA

15. Jesse likes variety. So, he is going to flip the coin until he sees every possible result at least once. So far, all 14 of his flips have resulted in a knight! What is the probability that his 15th flip also results in a knight?

    (A) 0.65   (B) 0.3   (C) 0.05   (D) 0.0016   (E) NOTA

16. Pratheek is running a game booth at a carnival at his local middle school (such a nice guy)! It costs 4 tickets to play the game and the middle schoolers win 1 ticket if they flip the coin and get a knight and 5 tickets if they flip the coin and get a dragon. What is the maximum number of tickets Pratheek can give the younglings for landing the coin on its side and still have the game be profitable for him?

    (A) 35   (B) 36   (C) 37   (D) 38   (E) NOTA

17. An important part of being a statistician is collecting accurate data. This means that the methods used to collect data must be carefully designed to eliminate as many sources of bias as possible. Romir is collecting data from a school and wants to utilize cluster sampling to do so. Which of the following descriptions uses cluster sampling to select students?

    (A) Splitting the school by grade and randomly selecting 10 students from each grade
    (B) Selecting the first student to get to school on a random day and selecting every 5th student after that
    (C) Assigning every student a unique id number and using a random number generator to select students
    (D) Going into the first classroom he sees and selecting a random number of students
    (E) NOTA

18. Little did the school know, but Romir is actually an evil scientist. He chose 500 random students from the school and is going to inject each one with a chemical. Romir wants to randomly assign each student to receive chemical A or chemical B and will also randomly give a high dosage, medium dosage, or low dosage of the chosen chemical. Being a good scientist, Romir made sure to also include a control group. How many different treatments will Romir be subjecting the students to?

    (A) 6   (B) 7   (C) 8   (D) 9   (E) NOTA

19. Howard is an amazing blackjack player who somehow never busts! Howard is so confident in his ability that he always goes all in. He mistakenly believes that since he has been winning so much recently, he is going to keep winning. What is the name of the logical fallacy he has fallen for?

    (A) Card Counters Fallacy   (B) Blackjack Fallacy   (C) Hot Hand Fallacy   (D) Gambler's Fallacy   (E) NOTA

Use the following information for the next 3 questions. Unless specified otherwise, each question is independent of one another so the situations in one question will not affect the situations in another.

The following table represents the possible outcomes of an unfair die and the probability of each outcome.

Data						
Result	1	2	3	4	5	6
Probability	0.1	0.2	0.3	0.2	0.1	x

20. What is the missing value, x?

(A) 0.4   (B) 0.3   (C) 0.2   (D) 0.1   (E) NOTA

21. Using the missing value found in the previous problem, what is the expected value and standard deviation of the die rolls, each rounded to the thousandths place? Answers are in the form (expected value, standard deviation).

(A) (3.300, 2.010)   (B) (2.010, 3.300)   (C) (3.300, 1.418)   (D) (1.418, 3.300)   (E) NOTA

22. Let X be a random variable that represents the die roll. What is the expected value of X^2, rounded to the thousandths place?

(A) 2.010   (B) 3.300   (C) 10.890   (D) 12.900   (E) NOTA

23. The FAMAT delegates love merch. We love money too. So, to get money, we will be selling merch! Every piece of merch will be randomly assigned a coolness factor. The coolness factor is an arbitrary number that follows a normal distribution with mean of 100 and standard deviation of 20. Using the empirical rule, what is the probability that a randomly selected piece of merch has a coolness factor greater than 120 given that it has coolness factor less than 160, rounded to three decimal places.

(A) 0.997   (B) 0.680   (C) 0.341   (D) 0.340   (E) NOTA

24. How many of the following are resistant measures?
I Mean   II Mean   III Mode   IV Standard Deviation   V Range   VI IQR

(A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

25. Which of the following statements is true about using an approximately normal distribution to represent a sampling distribution of a sample mean?

(A) You may do it if np and nq are both at least 10
(B) You may only do it if the population you are sampling from is normally distributed
(C) You may do it any time the sample is chosen randomly
(D) You may do it if the sample size is at least 30
(E) NOTA

Use the following information for the next 2 questions. Unless specified otherwise, each question is independent of one another so the situations in one question will not affect the situations in another.

The following two-way frequency table represents the favorite anime (One-Piece or Pokemon) of a random sample of 240 people, split by age (old or young).

	One-Piece	Pokemon	Total
Young	21	39	60
Old	135	45	180
Total	156	84	240

26. What is the probability that randomly selected person out of the 240 in the sample prefers Pokemon given that they fall into the old age bracket?

(A) $\frac{3}{4}$ (B) $\frac{13}{20}$ (C) $\frac{7}{20}$ (D) $\frac{15}{28}$ (E) NOTA

27. Based on the information provided in the table, which of the following is true?

(A) Favorite anime is independent of age   (B) Favorite anime is not independent of age
(C) One-Piece is the world's favorite anime   (D) More old people than young people like anime
(E) NOTA

28. Dr. DR believes that drawing the stupid picture will make solving any problem easy. To test this theory, he is going to construct a 95% confidence interval for the true proportion of times he has drawn the stupid picture and gotten a correct answer. Using the fact that he drew pictures for 100 problems and got 80 of them right, how large should his sample be to ensure that the width of his confidence interval is at most 0.15. You may assume that all appropriate conditions are met.

(A) 77   (B) 76   (C) 66   (D) 87   (E) NOTA

29. Jesse wants to know his approval rating. So, he took a random sample of 277 Mu Alpha Theta members. The following table displays the data he collected:

	Mu	Alpha	Theta
Approve	10	10	10
Neutral	23	41	68
Disapprove	16	84	15

If he were to do a chi-square test for independence, which of the following would give you the corrected expected cell count for the number of Alphas that approve of him, rounded to the thousandths place?

(A) 44.3177   (B) 10.072   (C) 10.000   (D) 0.036   (E) NOTA

30. Did you know that Alejandro's favorite country is Spain? Alejandro is also very deranged, so he wants to find the number of distinct derangements of the letters of the word "SPAIN". Assuming Alejandro correctly computes this number, what does he get?

(A) 44   (B) 73   (C) 119   (D) 120   (E) NOTA