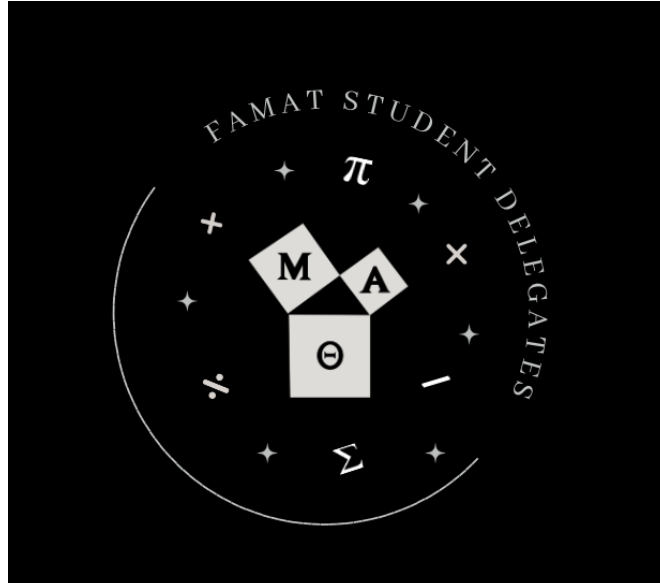# 2023 Student Delegate Invitational

Alpha Individual

- You may not leave the zoom until the testing time of one hour is complete.
- No calculators or any other electronic devices are permitted.
- Sunglasses and hats are not to be worn.
- At the end of the testing session, turn in only the google form. You may use whatever scrap paper you wish, provided it is blank.
- The scoring will be 5 points for each question answered correctly, 1 point for each question left blank, and 0 points for each missed question. Ties will be broken using the sudden death method. Ask your proctor if you do not know what this is.
- Answers will be posted at the end of the test.
- If you believe that none of the given answers are correct, choose answer choice E, for none of the above.
- If you believe that multiple answer choices are correct, choose one and file a dispute after the test.
- Unless a question asks for an approximation or a rounded answer, give the exact answer.
- You will have 60 minutes to complete this test, with warnings given when you have 15 minutes, 5 minutes, and 1 minute left.

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. The reference angle 140° is not coterminal with which angle?
   (A) 3020   (B) -2380   (C) 5180   (D) -6880   (E) NOTA

2. What is the dot product of the vectors $< 1, 3, 8 >$ and $< 2, 4, 7 >$?
   (A) 70   (B) 0   (C) 12   (D) 74   (E) NOTA

3. Vector $\mathbf{r} = 4i - 7j + 2k$, $\mathbf{s} = 2i + 5j + 3k$, and $\mathbf{t} = 4i - j - k$. What is $(2\mathbf{t} \cdot \mathbf{s}) \times r$?
   (A) -21   (B) -27   (C) 0   (D) 10   (E) NOTA

4. Find the sum of the eigenvalues of the matrix:

$$\begin{bmatrix} 1082 & 4623 \\ 1342 & 1823 \end{bmatrix}$$

   (A) 1873   (B) 1000   (C) 2900   (D) 2905   (E) NOTA

5. What is $\tan(\cos^{-1}(\frac{x}{4}))$?
   (A) $\frac{x}{\sqrt{16-x^2}}$ (B) $\frac{x}{4}$ (C) $\frac{\sqrt{16-x^2}}{x}$ (D) $\frac{4}{x}$ (E) NOTA

6. What is $\sin(2023\pi)$
   (A) 0   (B) 1   (C) 2023   (D) -1   (E) NOTA

7. Esteban is buying Maria flowers, and he notices an especially pretty flower in the bouquet. It's in the shape of the polar curve $r = 2\cos^2(x) - 1$. How many petals does it have?
   (A) 1   (B) 2   (C) 3   (D) 4   (E) NOTA

8. How many permutations of $SUKEERTH$ have the E's right next to each other?
   (A) 1380   (B) 40320   (C) 20160   (D) 5040   (E) NOTA

9. Identify this curve: $r = 3 + 2sin\theta$.
   (A) Inner-loop Limacon   (B) Cardioid   (C) Dimpled Limacon   (D) Rose Curve   (E) NOTA

10. What is the period of $\sin(2x) + \cos(x)$
    (A) π (B) 2π (C) 3π (D) $\frac{1}{2}\pi$ (E) NOTA

11. If $z = \sqrt{3} + i$, find z^6.
    (A) -64   (B) 64   (C) -64i   (D) $\sqrt{3} + i$ (E) NOTA

12. Find the average of the sum of $n\sin(n°)$ where n = 2, 4, 6, 8, 10, 12 ... 180
    (A) $\cot 1°$ (B) $\sin 180°$ (C) $\cos 184°$ (D) $\tan 1°$ (E) NOTA

13. What is the amplitude of $8\sin(\theta)\cos(\theta) + 3\cos(2\theta)$?
    (A) 11   (B) 5   (C) $\sqrt{73}$ (D) $\sqrt{55}$ (E) NOTA

14. The function $y = 3x + 8$ intersects the quadratic $y = 184x^2 + 138x$ at two different points, what is the slope of the line that passes through these two points.

    (A) 9   (B) 3   (C) 7   (D) 4   (E) NOTA

15. The Real Nate Caves is pushing a box in a straight line from the point (12, 14, 21) to (3, 31, 23) (he's standing on a very wonky plane). However, he applies the force in the direction $\langle 1, 2, 3 \rangle$ (he's a little disoriented). What's the magnitude of the force that actually gets applied to the box?

    (A) $\frac{31}{\sqrt{374}}$ (B) $\frac{31}{374}$ (C) $\sqrt{14}$ (D) 0   (E) NOTA

16. What is the volume of a tetrahedron with vertices (8, 2, 5), (10, 5, 7), (8, 8, 4), and (4, -1, 10)?

    (A) 114   (B) 30   (C) 19   (D) 5   (E) NOTA

17. Jonathan loves to play Fortnite. Everyday he plays Fortnite for 100 minutes, how many milliseconds of fortnite does he play in one year?

    (A) 2400   (B) 240   (C) 6000000   (D) 2190000   (E) NOTA

18. Sukeerth's favorite conic is the conic $x^2 + 12xy + 16y^2 + 8x + 5y = 6$. When rotating it by $\theta°$ he notices the conic takes the new shape $Ax^2 + Cy^2 + Dx + Ey + F = 0$. What is $A + C$?

    (A) 18   (B) 0   (C) 24   (D) 17   (E) NOTA

19. Deetya, Isabella, and Nishini are standing on the points (5, 21, 6), (8, 19, -2), and (3, 9, 8), respectively. What is the equation of the plane that contains all three of them?

    (A) $10x + 1y + 4z = 71$ (B) $-100x + 10y - 40z = 700$
    (C) $10x - 1y + 4z = 53$ (D) $100x - 10y + 40y = 580$
    (E) NOTA

20. Shravan is in love with cylinders. Everywhere he goes, he only thinks about cylinders. On his 16th birthday, he learned about cylinder coordinates. What is the cylindrical coordinate representation of $(16, 16, 16)$

    (A) $16, 45°, 16$ (B) $16\sqrt{2}, 45°, 16$ (C) $16\sqrt{2}, 45°, 16\sqrt{2}$ (D) $16\sqrt{3}, 60°, 16$ (E) NOTA

21. Find the equation of the conic section that contains the point (3, 2) and has its focus at (-2, 14), with a corresponding directrix of $y = -24$.

    (A) $3x^2 + 224 + 4y^2 + 16y - 260 = 0$ (B) $3x^2 - 260x + 4y^2 + 16y + 224 = 0$
    (C) $3x^2 + 16x + 4y^2 - 160y + 224 = 0$ (D) $4x^2 + 16x + 3y^2 - 160y + 224 = 0$
    (E) NOTA

22. Jaansi was in art class and decided to draw a rose. For some reason, she realized that the rose exactly resembled the graph of $r = 4sin(8\theta)$. How many petals did her rose have?

    (A) 4   (B) 8   (C) 2   (D) 12   (E) NOTA

23. What is the rank of the following matrix: $\begin{bmatrix} -1 & -3 \\ 3 & -3 \\ -3 & -3 \\ 2 & 0 \end{bmatrix}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

24. Jaansi and Sukeerth are both running away from Jesse. Jaansi runs along the line $y = 3x + 8$. Sukeerth runs along the line $y = 2x + 4$. What is the tangent of the smaller angle between both of these lines?

(A) $\frac{1}{7}$ (B) $\frac{2}{3}$ (C) $\frac{4}{7}$ (D) $\frac{1}{8}$ (E) NOTA

25. The Witch of Agnesi is a famous curve studied by Maria Gaetana Agnesi, the first woman to write a mathematics handbook and the first woman appointed as a mathematics professor at a university. The curve has many applications in probability theory and approximations of functions by polynomials, but this isn't a statistics or calculus test, so all we'll ask you to do is convert these two parametric equations to a single equation of y in terms of x (and a).

$$y = 2a\sin^2(\theta)$$
$$x = 2a\cot(\theta)$$

(A) $\frac{2\sqrt{8a^3-1}}{x^2}$ (B) $\frac{32a^3}{x^2}$ (C) $\frac{8}{x+2a^2}$ (D) $\frac{8a^3}{x^2+4a^2}$ (E) NOTA

26. If $\sin(x+y) = 0.6$ and $\sin(x-y) = 0.4$. What is $100\sin(x)\cos(x)$

(A) 100 (B) 50 (C) 25 (D) 75 (E) NOTA

27. Commander G is not only a top statistician, he is also a world class linguist! In addition to being fluent in English, he can also speak Canadian and Spanish. On any given day, Commander G will speak in a random combination of these three languages (always speaking in at least one of them). Of the 100 days he was observed, Commander G spoke only English on 34 of them, only Spanish on 22 of them, and only Canadian on 8 of them. He spoke all three on 10 days. If he spoke Canadian on a total of 25 days, how many days did he speak only English and Spanish?

(A) 19 (B) 21 (C) 24 (D) 26 (E) NOTA

28. Let's say p for $\cos(3p\sin(x)) = \sin(3p\cos(x))$ where x is between 0 and 2π. Let's say a is the number of minutes in a week? What is $a \cdot p$

(A) 1800π (B) 1380π (C) 1408π (D) 1520π (E) NOTA

29. Romir loves to play minecraft. He has been playing so much minecraft that his friends locked his computer. The password to the computer is the solution to the equation $y = \tan^2(20°) + \tan^2(40°) + \tan^2(80°)$ What is y?

(A) 38 (B) 37 (C) 31 (D) 33 (E) NOTA

30. Congratulations on finishing the test! Here is an easy one, how many distinct permutations does the word FAMAT have.

(A) 360 (B) 720 (C) 120 (D) 60 (E) NOTA