For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Evaluate $\lim_{x \to 2022} \frac{2022}{x}$

   (A) 0 (B) 1 (C) 2022 (D) DNE (E) NOTA

2. Find the derivative of $x^2 + 3x + 5$ at $x = 2$

   (A) 2 (B) 5 (C) 7 (D) 15 (E) NOTA

3. For what value of a does the line tangent to $y = x^2 + ax$ at the point $(0, 0)$ have a slope of 3?

   (A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 3 (E) NOTA

4. Find the derivative of $x^2 \sin(x)$

   (A) $2x\cos(x)$ (B) $x^2\cos(x)$ (C) $2x\sin(x)$ (D) $2x\sin(x) + x^2 e^x \cos(x)$ (E) NOTA

5. Sharvaa has 10 ft of fencing and wants to make a pen to enclose his kittens. If the maximum area he can enclose is A ft^2, find $\lfloor 10A \rfloor$

   (A) 62 (B) 79 (C) 80 (D) 250 (E) NOTA

6. What is the maximum value of $1 - x^{2/3}$ on $[-1, 2]$

   (A) 0 (B) $1 - 2^{-2/3}$ (C) 1 (D) $1 - 2^{2/3}$ (E) NOTA

7. Find the area bound by the curves $f(x) = x^2$ and $g(x) = 4 - x^2$

   (A) $\frac{8\sqrt{2}}{3}$ (B) $\frac{16\sqrt{2}}{3}$ (C) $\frac{32}{3}$ (D) $\frac{64}{3}$ (E) NOTA

8. Himal has two numbers. If the sum of the two numbers is 1, find the maximum value of the sum of the reciprocals of these two numbers.

   (A) 1 (B) 2 (C) 4 (D) 8 (E) NOTA

9. Find the derivative of $(\sin(x))^x$ at $x = \frac{\pi}{6}$

   (A) $\left(\frac{1}{2}\right)^{\frac{\pi}{6}} \left(\frac{\pi\sqrt{3}}{6} - \ln(2)\right)$ (B) $\left(\frac{\sqrt{3}}{2}\right)^{\frac{\pi}{6}} \left(\frac{\pi\sqrt{3}}{6} + \ln\left(\frac{\sqrt{3}}{2}\right)\right)$

   (C) $\frac{\pi\sqrt{3}}{6} - \ln(2)$ (D) $\frac{\pi\sqrt{3}}{6} + \ln\left(\frac{\sqrt{3}}{2}\right)$ (E) NOTA

10. How many inflection points does the function $f(x) = x^4 - 5x^3 + 3x^2$ have?

    (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

11. Evaluate the following integral:

$$\frac{d}{dx} \int_0^{x^3} \tan(2t^2) dt$$

    (A) $\tan(2x^6)$ (B) $3x^2 \tan(2x^6)$ (C) $12x^5 \sec^2(2x^6)$ (D) No Closed Form (E) NOTA

12. Approximate $\sqrt{26}$ using differentials and $f(x) = \sqrt{x}$ as x goes from 25 to 26

(A) 5.09 (B) 5.099 (C) 5.1 (D) 5.11 (E) NOTA

13. Determine the sum of all c which satisfy the conclusions of the Mean Value Theorem for the following function:

$$f(x) = \sin(x) \text{ on } [0, 2\pi]$$

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{4}$ (C) π (D) 2π (E) NOTA

14. Jesse has skipped nationals practice to fish! He is in a boat with an anchor dropped to the bottom of the lake that is attached to a rope that is 10 feet long. If this anchor remains fixed and there is slack within the rope, the region his boat can drift in has an area of A. Find the rate of change of this area when the water level is at 6 ft and is dropping at a rate of 1 ft/hr

(A) $12\pi\frac{\text{ft}^2}{\text{hr}}$ (B) $12\frac{\text{ft}^2}{\text{hr}}$ (C) $24\pi\frac{\text{ft}^2}{\text{hr}}$ (D) $24\frac{\text{ft}^2}{\text{hr}}$ (E) NOTA

15. The rate of change in the amount of weight Tanmay can deadlift is modeled by the differential equation below:

$$\frac{dW}{dt} = \frac{W}{t} + Wt$$

W is how much weight he can lift in pounds and t is time in days. Given that $W(1) = 20$, what is $W(29)$?

(A) $69e^{420}$ (B) $360e^{420}$ (C) $580e^{420}$ (D) $640e^{420}$ (E) NOTA

16. Find the arclength of $\sqrt{1-x^2}$ from $x = 0$ to $x = \frac{\sqrt{3}}{2}$

(A) $\frac{1}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\pi}{3}$ (E) NOTA

17. Evaluate the following integral:

$$\int_{\frac{3}{5}}^{\frac{4}{5}} \frac{1}{x^2\sqrt{1-x^2}}dx$$

(A) $\frac{3}{5}$ (B) $\frac{7}{12}$ (C) $\frac{4}{5}$ (D) $\frac{11}{12}$ (E) NOTA

18. Find the area that is outside $r = \cos(\theta)$, but is inside $r = 1$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π (E) NOTA

19. Use Euler's Method with a step size of 1 to find $y(2)$ given $y(0) = 1$ and

$$\frac{dy}{dx} = x + y$$

(A) 5 (B) 7 (C) 9 (D) 11 (E) NOTA

20. Determine the convergence of the series below:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n + \frac{1}{n}}$$

(A) Diverges (B) Converges Conditionally (C) Converges Absolutely (D) Bifurcates (E) NOTA

21. In higher dimensions, mathematicians will sometimes use a tangent plane approximation instead of a tangent line approximation to estimate values on a surface. Find the plane tangent to $x^2+y^2+z^2=1$ at the point $(\frac{3}{13},\frac{4}{13},\frac{12}{13})$ and use it to estimate the value of z when $x=\frac{1}{3}$ and $y=\frac{1}{4}$.

(A) $\frac{11}{12}$ (B) $\frac{11}{13}$ (C) $\frac{\sqrt{6}}{12}$ (D) $\frac{\sqrt{7}}{13}$ (E) NOTA

22. Evaluate: $\lim_{x\to 0}\frac{\sqrt{x^2+9}-\sqrt[3]{x^3+27}}{1-\cos(x)}$

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) NOTA

23. The antiderivative of $x^n e^x$ can be written in the form of $P_n(x)e^x+C$ where $P_n(x)$ is some polynomial of degree n and C is the constant of integration. Find the sum of the squares of the roots of $P_{10}(x)$.

(A) -80 (B) -20 (C) 20 (D) 80 (E) NOTA

24. Evaluate the following integral:

$$\int_0^1 (\ln(x))^4 x^4 dx$$

(A) $-\frac{3}{128}$ (B) $\frac{24}{3125}$ (C) $-\frac{24}{625}$ (D) $\frac{5}{1944}$ (E) NOTA

25. Find the slope of the tangent line of the hyperbola $7y^2-24xy+2y-11x+3=0$ at either of the vertices.

(A) $-\frac{7}{24}$ (B) $-\frac{4}{3}$ (C) 0 (D) $\frac{3}{4}$ (E) NOTA

26. The plane $y=c$ intersects the solid torus formed by rotating the region enclosed by $(x-2)^2+y^2=1$ around the y-axis. The cross-sectional area, A changes as c increases. Find $\frac{dA}{dt}$ when $c=-\frac{3}{5}$ and $\frac{dc}{dt}=1$

(A) 6π (B) 4π (C) 3π (D) π (E) NOTA

27. Taylor Swift is a Mastermind when it comes to differential equations, but today she has a headache and is stuck in a Lavender Haze. She knows some differential equations can be solved by guessing that the solution can be represented as a generalized power series and solving for the coefficients. However, when it comes to this Labyrinth of a question, everything just feels Bigger Than The Whole Sky to her. All she wishes is for an Anit-Hero to come along and help end what feels like The Great War of differential equations problems. Make Taylor Swift feel Bejeweled by solving the following differential equation given that $y(0)=1, y'(0)=0$ and assuming that $y=\sum_{n=0}^{\infty} a_n x^n$.

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$$

What is the value of a_6?

(A) $-\frac{1}{45}$ (B) $-\frac{1}{2304}$ (C) $\frac{1}{45}$ (D) $\frac{1}{2304}$ (E) NOTA

28. While Taylor Would've, Could've, Should've solved the previous Question...?, she was too busy daydreaming about Snow On The Beach to think and decided to just guess some Sweet Nothing instead using her knowledge of Taylor series. Consider Taylor's guess below for y:

$$y(x) = -x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \ldots + (-1)^n \frac{1}{n}x^n \ldots$$

Taylor Swift tried to compute the value $y(k)$ for some real value k, but her computer had a Glitch and reported k was outside the interval of convergence of $y(x)$. Feeling like it was the Karma she deserved for guessing, Taylor Swift went to Paris for validation, but was just told "You're On Your Own, Kid" by locals. As it approaches 3am and starts to storm, Taylor feels as if society will Maroon her in this Midnight Rain like she has committed some High Infidelity. All she wishes is for some mathematical vigilante to come help explain the error to her. Dear Reader, give Taylor the Vigilante **** she deserves and find the interval k could not possibly have been in.

(A) $(-1, 1)$ (B) $[-1, 1)$ (C) $(-1, 1]$ (D) $[-1, 1]$ (E) NOTA

29. Ritvik decides to go for a boat ride. He is traveling in the xy plane along some function $y(x)$ where x and $y(x)$ are given in meters, and his goal is to get from the point $(0, 0)$ to $(1, 1)$. The slope of the tangent line to $y(x)$ at $(0, 0)$ is 0, and the slope of the tangent line to $y(x)$ at $(1, 1)$ is 2. His speed as he travels along the curve is given by $\frac{1}{y''(x)}\frac{\text{m}}{\text{s}}$. Find how long it takes for him to get from $(0, 0)$ to $(1, 1)$. All answers are given in seconds.

(A) $\frac{\ln(2)}{4} + 2\sqrt{2}$ (B) $\frac{\ln(\sqrt{2}+1)}{2} + \frac{\sqrt{2}}{2}$ (C) $\frac{\ln(3)}{6} + 2\sqrt{3}$ (D) $\frac{\ln(\sqrt{5}+2)}{2} + \sqrt{5}$ (E) NOTA

30. Consider $g_0(x)$ defined below:

$$g_0(x) = \begin{cases} 0 & \text{if } x \leq -\frac{5}{4} \\ 1 & \text{if } -\frac{5}{4} < x < \frac{5}{4} \\ 0 & \text{if } x \geq \frac{5}{4} \end{cases}$$

$g_k(x)$ is defined below for positive integer k:

$$g_k(x) = \frac{1}{k}\int_{x-\frac{k}{2}}^{x+\frac{k}{2}} g_{k-1}(t)dt$$

$g_k(0) = 1$ for all $k \leq n$ where n is an integer. Find $n + 1$. (The first k value for which $g_k(0) \neq 1$)

(A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA