Assume that all angles are in radians unless otherwise stated. For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Compute $\sin(\frac{\pi}{6})$.

   (A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) NOTA

2. What is $<2,3> \cdot <4,5>$?

   (A) 5 (B) 14 (C) 23 (D) 26 (E) NOTA

3. What is $<1,2,3> \times <4,5,6>$?

   (A) $<-3,6,-3>$ (B) $<-3,-6,-3>$ (C) 21 (D) 32 (E) NOTA

4. Sharvaa wants to maximize the area of his pen so that he can fit all of his kittens in it. He has 20 feet of fencing. What is the maximum area he can enclose within his pen?

   (A) 10 (B) 20 (C) 25 (D) 40 (E) NOTA

5. $110°$ is the reference angle for which of the following angles?

   (A) $70°$ (B) $700°$ (C) $1100°$ (D) $2270°$ (E) NOTA

6. Find the number of solutions on the interval $(-2\pi, 2\pi]$ to the equation $\cos(2x) = \cos(x)$

   (A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA

7. What is the amplitude of the function $f(x) = 3\sin(x) + 4\cos(x)$

   (A) $\sqrt{7}$ (B) 3 (C) 4 (D) 5 (E) NOTA

8. What is the area of the ellipse $9x^2 - 18x + 4y^2 + 8y - 11 = 0$?

   (A) π (B) 6π (C) 12π (D) 36π (E) NOTA

9. A parabola has a directrix with equation $x = 2$ and focus at the point $(4, 6)$. What is the sum of the abscissa of the endpoints of the latus rectum?

   (A) 6 (B) 8 (C) 10 (D) 12 (E) NOTA

10. What is the determinant of the following matrix?

$$\begin{bmatrix} \cos(\frac{\pi}{3}) & \sin(\frac{\pi}{3}) \\ -\sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) \end{bmatrix}$$

    (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{2}$ (E) NOTA

11. Find $\sin(\frac{5\pi}{12})\sin(\frac{\pi}{12})$

    (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{\sqrt{3}}{8}$ (D) $\frac{\sqrt{3}}{4}$ (E) NOTA

12. What is the polar form of $z = -2 + 2\sqrt{3}i$

(A) $4e^{\frac{11\pi}{6}}$ (B) $4(\cos\frac{11\pi}{6} + i\sin(\frac{11\pi}{6}))$ (C) $4(\cos\frac{2\pi}{3} + i\sin(\frac{2\pi}{3}))$ (D) $4e^{\frac{2\pi}{3}}$ (E) NOTA

13. If $f(z) = \dfrac{1}{1-z}\ln\dfrac{1}{1-z}$, which of the following is a value of $f(e^{\frac{i\pi}{6}})$? (NOTE: Disputes saying there are multiple answers to this question will not be accepted. Pick the answer choice that is one of the solutions).

(A) 1 (B) $\dfrac{i\pi e^{\frac{i\pi}{6}}}{6}$ (C) $\dfrac{i\pi e^{\frac{i\pi}{3}}}{3}$ (D) $-\dfrac{\pi}{2}$ (E) NOTA

14. A point α with coordinates $(7, 5)$ is reflect across the line $y = -7x + 4$. What are the coordinates of α after this reflection?

(A) $(-7, 5)$ (B) $(-7, 3)$ (C) $(5, 7)$ (D) $(-5, 7)$ (E) NOTA

15. What is/are the eigenvalue(s) of the following matrix?

$$\begin{bmatrix} \cos(\frac{\pi}{3}) & \sin(\frac{\pi}{3}) \\ -\sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) \end{bmatrix}$$

(A) 0 (B) $e^{\frac{i\pi}{6}}$, $-e^{\frac{i\pi}{6}}$ (C) $e^{\frac{i\pi}{4}}$, $-e^{\frac{i\pi}{4}}$ (D) $e^{\frac{i\pi}{3}}$, $-e^{\frac{i\pi}{3}}$ (E) NOTA

16. The equation of a conic section is given as $r = \dfrac{13}{2 + 4\cos\theta}$. What kind of conic is it?

(A) Hyperbola (B) Parabola (C) Circle (D) Ellipse (E) NOTA

Use the following information for questions 17, 18, and 19:
$\vec{v_1} =< 1, 2, 2 >$, $\vec{v_2} =< 4, 2, 4 >$, $\vec{v_3} =< 3, 4, 12 >$

17. Find the volume of the parallelepiped defined by $\vec{v_1}, \vec{v_2}$, and $\vec{v_3}$

(A) 32 (B) 44 (C) 48 (D) 66 (E) NOTA

18. Find the component of $\vec{v_2}$ perpendicular to $\vec{v_1}$. Call this vector $\vec{u_2}$ for Question 19

(A) $< \frac{16}{9}, \frac{32}{9}, -\frac{32}{9} >$ (B) $< -\frac{64}{9}, \frac{32}{9}, \frac{64}{9} >$ (C) $< \frac{20}{9}, -\frac{14}{9}, \frac{4}{9} >$ (D) $< -\frac{7}{9}, \frac{2}{9}, \frac{2}{9} >$ (E) NOTA

19. Find the component of $\vec{v_3}$ that is perpendicular to $\vec{v_1}$ and $\vec{u_2}$

(A) $< -4, 4, 6 >$ (B) $< 4, -6, 4 >$ (C) $< -4, -4, 6 >$ (D) $< 4, 4, -6 >$ (E) NOTA

20. Square $ABCD$ has side length 2 and lies in the xy-plane. Point A' lies two units above point A and the xy-plane. Points B', C', and D' are above B, C, and D respectively. Point M is the midpoint of line segment $A'D'$. Find $\cos(\angle BMC)$

(A) $\dfrac{1}{2}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{\sqrt{3}}{2}$ (D) $\dfrac{11}{12}$ (E) NOTA

21. Find the sum of the 2022^{nd} roots of $r^{2022} - 4r^2 + 2r - 2 = 0$

(A) 4044 (B) 3021 (C) 2022 (D) 0 (E) NOTA

22. Himal takes a survey of the 400-person Buccholz math team about which pies they like (Himal includes herself in the survey). 120 of them say they like apple pie, 170 of them say they like cherry pie, and 180 of them say that they like pumpkin pie. 30 people like all 3 flavors, 80 people like Apple and Cherry, 60 people like Cherry and Pumpkin, and 50 people like Apple and Pumpkin. What is the minimum number of Buccholz math students that would need to go to the pie store to guarantee that there was at least one person at the store that liked Cherry pies? (Assume that if someone did not say they liked a certain flavor of pie then they *do not* like that flavor of pie).

(A) 61 (B) 141 (C) 171 (D) 231 (E) NOTA

23. Ariana Grande decided to quit the music business and become a mathematician. Because her song 34+35 was such a hit, she decided to write the following which she calls the Great Grande: $\sqrt{(\sqrt{34}+\sqrt{35}+\sqrt{69})(\sqrt{34}+\sqrt{35}-\sqrt{69})(\sqrt{34}-\sqrt{35}+\sqrt{69})(-\sqrt{34}+\sqrt{35}+\sqrt{69})}$. Which of the following is equivalent to the Great Grande?

(A) $\sqrt{1190}$ (B) 34.5 (C) 69 (D) $2\sqrt{1190}$ (E) NOTA

24. Find the sum of the period, amplitude, vertical shift, and phase shift of $y = 18\cos(\frac{16x}{\pi} - 2) - 12$.

(A) $\frac{\pi^2}{8} + 4$ (B) $\frac{\pi^2}{8} + 28$ (C) $\frac{\pi^2}{8} + \frac{\pi}{8} + 6$ (D) $\frac{\pi^2}{8} + \frac{\pi}{8} + 30$ (E) NOTA

25. Find the area of the region enclosed by $x^2 + xy + y^2 = 1$.

(A) π (B) $\frac{2\pi}{\sqrt{3}}$ (C) 2π (D) $\sqrt{3}\pi$ (E) NOTA

26. The student delegates are hosting an informational session and they are expecting X students to show up. Webmaster Jesse is trying to create breakout rooms and assign students to them so students are distributed as evenly as possible. If Jesse opens 13 breakout rooms, there will be 6 rooms with an extra student. If he opens 17 breakout rooms, there will be 10 rooms with an extra student. If he opens 23 breakout rooms, there will be 16 rooms with an extra person. What is the sum of the digits of the smallest possible value of X given that it is a positive integer?

(A) 18 (B) 19 (C) 20 (D) 21 (E) NOTA

27. Given that a conic with the equation $Ax + Bxy + Cy + Dx + Ey + F = 0$ is rotated an angle θ and ends with the new equation $A'(x')^2 + B'x'y' + C'y'^2 + D'x' + E'y' + F' = 0$, which of the following statements must be true?

I $B = B'$
II $F = F'$
III $A + C = A' + C'$
IV $D + E = D' + E'$
V $B^2 + 4AC = (B')^2 + 4A'C'$

(A) I, II (B) III, IV (C) II, III, V (D) I, IV, V (E) NOTA

28. Given $a^2 + b^2 + c^2 = 2$, what is the maximum value of $2a + 3b + 3c$?

(A) 44 (B) 46 (C) 48 (D) 50 (E) NOTA

29. Compute the following sum:

$$\sum_{n=0}^{\infty} \frac{1}{(3n)!}$$

Hint: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

(A) $\frac{e}{3}$ (B) $\frac{\sqrt{e}+\sqrt{3}}{3\sqrt{e}}$ (C) $\frac{\sqrt{e}+2\cos(\frac{1}{2})}{3\sqrt{e}}$ (D) $\frac{e^{\frac{3}{2}}+2\cos(\frac{\sqrt{3}}{2})}{3\sqrt{e}}$ (E) NOTA

30. As part of their yearly fundraising campaign, the student delegates have launched a brand new brand of cool clothes guaranteed to increase anyone's rizz factor! Within the clothing brand, the delegates are launching the following clothing lines: hats, glasses, shirts, pants, and shoes. All merch and the relevant details are listed in the table below.

Item List

Product Name	Clothing Line	Price	Rizz Points
Bucket Hat	Hat	$10	20
Top Hat	Hat	$5	15
Propeller Hat	Hat	$13	26
Sunglasses	Glasses	$20	5
Nerd Glasses	Glasses	$20	-5
Student Delegate Shirt	Shirt	$20	1000
MAO Logo Shirt	Shirt	$25	1000
Buchholz Championship Shirt	Shirt	$10	350
Jeans	Pants	$100	100000
Sweatpants	Pants	$40	800
Sandals	Shoes	$40	1000
High-Heels	Shoes	$15	600

Mr. Snow will give you up to $100 if you can find the most efficient outfit made completely out of student delegate merch. Efficiency is defined as rizz points divided by dollar spent, so getting 20 rizz points for $10 is more efficient than getting 21 rizz points for $11. Any outfit is deemed unfashionable (and therefore completely inefficient) if it does not include a shirt and a pair of pants and/or if it includes more than one product from the same clothing line. Assuming you have succeeded in your goal of getting money from Mr. Snow, how many rizz points have you gained from your brand new outfit?

(A) 1,400 (B) 1,405 (C) 1,415 (D) 100,000 (E) NOTA