

1. Evaluate the following limit:

$$\lim_{x \rightarrow \infty} \frac{x^6 + x^5 + 4x^7 + x^3 + 2}{x^7 + 5x^4 + 6x^2 + 3}$$

2. The unit-step function  $\mu(x)$  is defined below:

$$\begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases}$$

Evaluate the following limit:

$$\lim_{x \rightarrow 0} \mu(\mu(x^2))$$

3. The population of bunnies in Sharvaa's house can be modeled as a function of time as shown below:

$$P(t) = \frac{200}{1 + e^{-20t}}$$

As time goes on, this population approaches a certain size,  $L$ . Find  $L$ .

4. Evaluate:  $\lim_{x \rightarrow \infty} \sqrt[3]{2x^3 + 6x^2 - x} - \sqrt[3]{2x^3 - 6x^2 - x}$

5. Evaluate:  $\lim_{x \rightarrow -2.5} [x^3 [x^2 [x]]]$  where  $[x]$  is the greatest integer less than or equal to  $x$ .

6. Evaluate:  $\lim_{x \rightarrow 0} \frac{x^2}{\ln(\sin(x)/x)}$

7. Evaluate the following limit:

$$\lim_{n \rightarrow \infty} \left( \frac{3n}{2n} \right)^{1/n}$$

8. Let  $A_n$  denote the region bounded by  $x^{2n} + y^{2n} = xy$  in the first quadrant. Find  $\lim_{n \rightarrow \infty} A_n$ .

9. (#30 2022 Feb Regional Calc Indiv) A sequence  $\{a_n\}_{n=1}^{\infty}$  is defined by  $a_1 = \sqrt{2}$  and  $a_{n+1} = \sqrt{2 + a_n}$  for all  $n \geq 1$ . Evaluate:

$$\lim_{n \rightarrow \infty} 4^n (2 - a_n)$$

10. The Stolz-Cesaro theorem states that for two sequences  $\{a_n\}_{n \geq 1}$  and  $\{b_n\}_{n \geq 1}$ , if  $\{b_n\}$  is strictly monotone and divergent and  $\lim_{n \rightarrow \infty} \frac{a_{n+1} - a_n}{b_{n+1} - b_n} = L$ , then  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ . Evaluate the following limit in terms of  $k$  using the Stolz-Cesaro Theorem where  $k$  is an integer:

$$\lim_{n \rightarrow \infty} \frac{\sum_{j=1}^n j^k}{(n+1)^{k+1}}$$